Large induced subgraphs via triangulations and CMSO

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Abstract

We obtain an algorithmic meta-theorem for the following optimization problem. Let φ be a Counting Monadic Second Order Logic (CMSO) formula and $t \geq 0$ be an integer. For a given graph $G = (V, E)$, the task is to maximize $|X|$ subject to the following: there is a set $F \subseteq V$ such that $X \subseteq F$, the subgraph $G[F]$ induced by F is of treewidth at most t, and structure $(G[F], X)$ models φ, i.e. $(G[F], X) \models \varphi$. Special cases of this optimization problem are the following generic examples. Each of these special cases contains various problems as a special subcase:

- Maximum Induced Subgraph with $\leq \ell$ copies of $\mathcal{F}_m$-cycles, where for fixed nonnegative integers m and ℓ, the task is to find a maximum induced subgraph of a given graph with at most ℓ vertex-disjoint cycles of length $0 \pmod m$. For example, this encompasses the problems of finding a maximum induced forest or a maximum subgraph without even cycles.
- Minimum $\mathcal{F}$-Deletion, where for a fixed finite set of graphs $\mathcal{F}$ containing a planar graph, the task is to find a maximum induced subgraph of a given graph containing no graph from $\mathcal{F}$ as a minor. Examples of Minimum $\mathcal{F}$-Deletion are the problems of finding a minimum vertex cover or a minimum number of vertices required to delete from the graph to obtain an outerplanar graph.
- Independent $\mathcal{H}$-packing, where for a fixed finite set of connected graphs $\mathcal{H}$, the task is to find an induced subgraph F of a given graph with the maximum number of connected components, such that each connected component of F is isomorphic to some graph from $\mathcal{H}$. For example, the problem of finding a maximum induced matching or packing into nonadjacent triangles, are the special cases of this problem.

We give an algorithm solving the optimization problem on an n-vertex graph G in time $\mathcal{O}(|\Pi_G| \cdot n^{t+4} \cdot f(t, \varphi))$, where Π_G is the set of all potential maximal cliques in G and f is a function of t and φ only. We also show how similar running time can be obtained for the weighted version of the problem. Pipelined with known bounds on the number of potential maximal cliques, we derive a plethora of algorithmic consequences extending and subsuming many known results on algorithms for special graph classes and exact exponential algorithms.

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